190:

1) $a(t) = 12t^2 - 4$, $x(1) = 3$

a) At rest when $v(t) = 0$

$v(t) = \int (12t^2 - 4)\,dt$

$v(t) = 4t^3 - 4t + C$

$v(0) = 0$

$0 = C$

$v(t) = 4t^3 - 4t$

$v(t) = 0$

$4t^3 - 4t = 0$

$4t(t^2 - 1) = 0$

$t = 0, t = \pm 1$

$t = 0, t = 1$

b) $x(t) = \int (4t^3 - 4t)\,dt$

$x(t) = t^4 - 2t^2 + C$

$x(1) = 3$

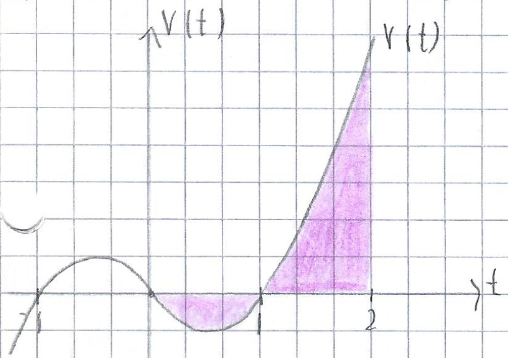
$1 - 2 + C = 3$

$-1 + C = 3$

$C = 4$

$x(t) = t^4 - 2t^2 + 4$

c) $v(t) = 4t^3 - 4t = 4t(t^2 - 1) = 4t(t+1)(t-1)$



Total Distance $= -\int_0^1 (4t^3 - 4t)\,dt + \int_1^2 (4t^3 - 4t)\,dt$

$= -\left[t^4 - 2t^2\right]_0^1 + \left[t^4 - 2t^2\right]_1^2$

$= -(1-2) + (16 - 8 - (1-2))$

$= 1 + 9$

$= 10$

2) $f(x) = \ln\left(\frac{x}{x-1}\right)$

a) $\frac{x}{x-1} > 0$ +   0   −   1   +   Domain: $(-\infty, 0) \cup (1, \infty)$

b) $f'(x) = \dfrac{(x-1)(1) - x(1)}{(x-1)^2} \div \dfrac{x}{x-1}$

$= \dfrac{-1}{(x-1)^2} \cdot \dfrac{x-1}{x}$

$= -\dfrac{1}{x(x-1)}$

$f'(-1) = \dfrac{-1}{-1(-2)} = -\dfrac{1}{2}$

c) $y = \ln\left(\dfrac{x}{x-1}\right)$

$x = \ln\left(\dfrac{y}{y-1}\right)$

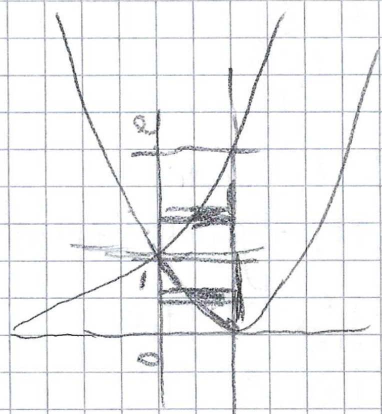
$e^x = \dfrac{y}{y-1}$

$(y-1)e^x = y$

$ye^x - e^x - y = 0$

$y(e^x - 1) = e^x$

$y = \dfrac{e^x}{e^x - 1} =$



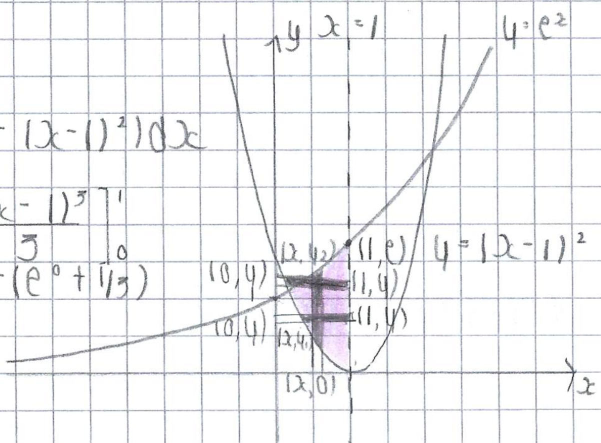
3) $y = e^x$, $y = (x-1)^2$

a) Area $R = \int_0^1 (e^x - (x-1)^2)\,dx$

$= \left[e^x - \dfrac{(x-1)^3}{3}\right]_0^1$

$= e^1 - 0 - (e^0 + 1/3)$

$= e - \dfrac{4}{3}$



$(1-\sqrt{y}, y)$

b) $R(x) = y_2 - 0 = e^x$

$r(x) = y_1 - 0 = (x-1)^2$

Volume $= \pi\int_0^1 [(e^x)^2 - (x-1)^4]\,dx$

$= \pi\left[\dfrac{e^{2x}}{2} - \dfrac{(x-1)^5}{5}\right]_0^1$

$= \pi\left[\left(\dfrac{e^2}{2} - 0\right) - \left(\dfrac{e^0}{2} + \dfrac{1}{5}\right)\right]$

$= \pi\left(\dfrac{e^2}{2} - \dfrac{1}{2} - \dfrac{1}{5}\right)$

$= \pi\left(\dfrac{5e^2 - 7}{10}\right)$

c) $y = e^x$ $\qquad$ $y = (x-1)^2$

$x = \ln y$ $\qquad$ $x - 1 = \pm\sqrt{y}$

$x = 1 \pm \sqrt{y}$

$R(x) = 1 - 0 = 1$ $\qquad$ $R(x) = 1 - 0 = 1$

$r(x) = x - 0 = 1 - \sqrt{y}$ $\qquad$ $r(x) = x - 0 = \ln y$

Volume $= \pi\int_0^1 1 - (1-\sqrt{y})^2\,dy + \pi\int_1^e 1 - (\ln y)^2\,dy$

$\frac{dr}{dt} = 0.04$ cm/second, $V = \frac{4}{3}\pi r^3$

a) We want $\frac{dV}{dt}$ when $r = 10$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(10)^2(0.04)$

$= 16\pi$ cm³/sec

b) $\frac{4}{3}\pi r^3 = 36\pi$

$r^3 = 36 \cdot \frac{3}{4}$

$r^3 = 27$

$r = 3$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(3)(0.04)$

$= 0.24\pi$

$\frac{dA}{dt} = 0.24\pi$ cm²/sec

c) $\frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$

$1 = 4\pi r^2$

$r^2 = \frac{1}{4\pi}$

$r = + \frac{1}{2\sqrt{\pi}} = \frac{\sqrt{\pi}}{2\pi}$ cm

5) $f(x) = \sin^2 x - \sin x$ for $0 \leq x \leq \frac{3\pi}{2}$

a) x intercepts: $y = 0$

$\sin^2 x - \sin x = 0$

$\sin x(\sin x - 1) = 0$

$\sin x = 0$, $\sin x - 1 = 0$

$x = 0, \pi$ $\quad$ $\sin x = 1$

$x = \frac{\pi}{2}$

$(0,0), \left(\frac{\pi}{2}, 0\right), (\pi, 0)$

b) $f'(x) = 2\sin x\cos x - \cos x$

$f'(x) = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$, $2\sin x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$f'(x)$: 0 $-$ $\frac{\pi}{6}$ $+$ $\frac{\pi}{2}$ $-$ $\frac{5\pi}{6}$ $+$ $\frac{3\pi}{2}$

Increasing on: $\left(\frac{\pi}{6}, \frac{\pi}{2}\right), \left(\frac{5\pi}{6}, \frac{3\pi}{2}\right)$

c) Rel. Min at $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$ since $f'(x)$ changes -ve to +ve at both points

Rel. Max at $x = \frac{\pi}{2}$ since $f'(x)$ changes sign from +ve to

$f\left(\frac{\pi}{6}\right) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} = -\frac{1}{4}$

$f\left(\frac{5\pi}{6}\right) =$

$f\left(\frac{\pi}{2}\right) = 1^2 - 1 = 0$

Check endpoint extrema:

$f(0) = 0$

$f\left(\frac{3\pi}{2}\right) = 2$

Abs. Max is 2

Abs. Min is $-\frac{1}{2}$

6) $f(x) = \frac{ax+b}{x^2-c}$

a) i) if symmetric: $\frac{a(-x)+b}{(-x)^2-c} = \frac{ax+b}{x^2-c}$

$-\frac{ax+b}{x^2-c} = \frac{ax+b}{x^2-c}$

ax must $= 0$

$a = 0$

ii) $\lim_{x \to 2^+} f(x) = +\infty \therefore x^2 - c = 0$

$2^2 - c = 0$

$c = 4$

$f(x) = \frac{b}{x^2-4}$

iii) $f'(1) = -2$

$f'(x) = \frac{(x^2-4)(0) - b(2x)}{(x^2-4)^2} = \frac{-2bx}{(x^2-4)^2}$

$-2 = \frac{-2b}{9}$

$-2b = -18$

$b = 9$

$a = 0, b = 9, c = 4$

b) Vertical Asymptote: $x^2 - 4 = 0$

$x^2 = 4$

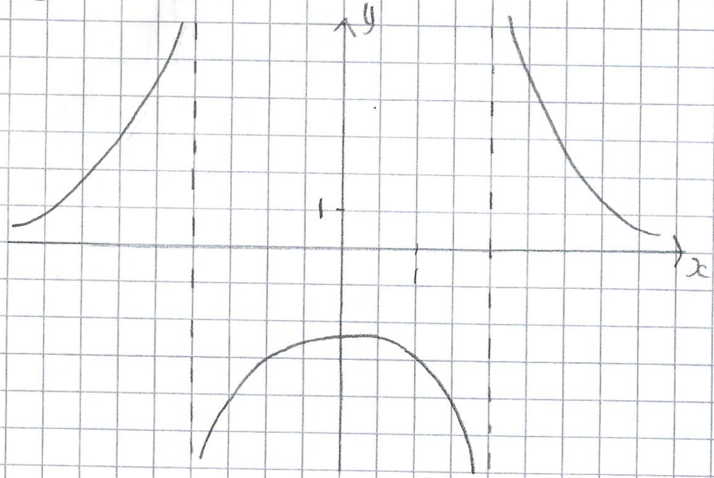
$x = \pm 2$

Horizontal Asymptote: $\lim_{x \to +\infty}\left(\frac{9}{x^2-4}\right) = 0$

$\lim_{x \to -\infty}\left(\frac{9}{x^2-4}\right) = 0$

x intercept: $\frac{9}{x^2-4} \neq 0$ $\therefore$ no x intercept

y intercept: $\frac{9}{0-4} = -\frac{9}{4}$



Turning Points:

$f'(x) = \frac{-18x}{(x^2-4)^2} = 0$

$x = 0, \quad x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2 \rightarrow$ vertical tangent

Turning point at $(0, -9/4)$ $\quad f'(x)$:

	+		−	
−2		0		2
	/		\	